Optimizing the Electric Grid:
Overcoming Nonconvexity and Scale

Richard Y. Zhang
Assistant Professor
Univ. of Illinois at Urbana-Champaign
ryz@illinois.edu
30% renewables

Source: Western Wind Integration Study
**Operator:** Human (+ machine assistance)

**Tool:** Intuition & Experience

---

**Operator:** Human-machine synergy

**Tool:** Mathematical optimization
Optimization for Power Systems

• **Challenges.** Nonconvexity and extreme scale

• Overcoming challenges using domain insight.

• **Summary and Outlook.**
Example. Maximum power point tracking

Easy: Greedy algorithm guaranteed to succeed
In optimization, greed is not always good

Hard: Greedy algorithm can fail, need global foresight

What makes 1 panel easy and 2 panels hard?
Issue 1. Nonconvexity.

Convex: Cut boundary at most twice
Issue 1. Nonconvexity.

Optimization: Finding extremal point of a shape.
If convex, then cannot get stuck at local min.
Goal 1. Guaranteed. Feasible & Near-Global

\[
\frac{\text{cost of global}}{\text{cost of candidate}} \geq 99\%
\]

Cost

Candidates

(Global)

(Guaranteed near-global)

Bound
How does work scale with size?

110 V

V1?

V2?

Income

Voltage 1

Voltage 2

Greedy (Linear): work-per-knob \times \text{number-of-knobs}

Foresight (Exponential): \text{work-per-knob}^{\text{number-of-knobs}}
Issue 2. Complexity. Time, Memory, Data

- **exp(n)**
- **n**
- **n^2**
- **n^3**

- time
- memory
- data

- number of parameters
- problem size

Hard

Easy?
Goal 2. Scalable. Linear time, Memory, Data

Explicating:

- **Linear time**: $O(n)$
- **Memory**: $O(n)$
- **Data**: $O(n)$

Scalable to millions (and billions?)

Number of parameters vs. problem size:

- $n$: Linear
- $n^2$: Quadratic
- $n^3$: Cubic
- $exp(n)$: Exponential
Optimization for Power Systems

• **Challenges.** Nonconvexity and extreme scale

• Overcoming challenges using domain insight.

• Summary and Outlook.
Optimal Power Flow on the Electric Grid

minimize cost of electricity over generator dispatch
subject to physics of electricity
reliability & security constraints

United States transmission grid
Source: FEMA
Nonconvexity: Equations of AC Electricity

minimize cost of electricity over generator dispatch
subject to physics of electricity
reliability & security constraints

\[ P_{av} \propto V_1 V_2 \sin \delta \]
Scale: Thousands to Tens of Millions

minimize cost of electricity over generator dispatch
subject to physics of electricity
       reliability & security constraints

Parameters
n > 15,000

Security Constraints
m > 90,000,000

Solve every 5 minutes
(to maintain supply = demand)
Existing inefficiency motivates new computational tools

Industry: Domain-specific heuristics
Approximations, decompositions, engineering judgment, reasonably acceptable solutions.
Fast but suboptimal. How suboptimal?

“Finding a good solution technique could potentially save tens of billions of dollars annually.”
-- FERC Report (2013)
Recent approaches based on convex relaxation / lift to higher dimen.
Quadratic Memory & Cubic time

\[ n = 3 \]
\[ n^2 \rightarrow 1 \text{ MB} \]
\[ n^3 \rightarrow 0.1 \text{ sec} \]

academic test case

\[ n = 120 \]
\[ n^2 \rightarrow 1.6 \text{ GB} \]
\[ n^3 \rightarrow 1.7 \text{ hr} \]

real-world

\[ n = 3,000 \]
\[ n^2 \rightarrow 1 \text{ TB} \]
\[ n^3 \rightarrow 3.2 \text{ years} \]

\[ n = 120 \times 40 \times 25 \]
\[ n = 3,000 \times 25 \]
\[ n = 3 \times 15,625 \]
Underlying tree-like graph structure: Decomposition approach

Rigorous notions: treewidth; tree decompositions; decoupled interactions.
Our Algorithm (Zhang & Lavaei 2020)

\[
\begin{align*}
\text{minimize} & \quad \text{cost of electricity over generator dispatch} \\
\text{subject to} & \quad \text{physics of electricity} \\
& \quad \text{reliability & security constraints}
\end{align*}
\]

Dualized Clique Tree Conversion with Auxiliary Variables

<table>
<thead>
<tr>
<th>Worst-case</th>
<th>Empirical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time: (O(n^{1.5}))</td>
<td>Time: (O(n))</td>
</tr>
<tr>
<td>Memory: (O(n))</td>
<td>Memory: (O(n))</td>
</tr>
</tbody>
</table>
In Practice: European Power System Model

Parameters

\( n = 13,659 \)

Security Constraints

\( m = 40,975 \)

\[ \text{Source: Cédric Josz, Stéphane Fliscounakis, Jean Maeght, Patrick Panciatici (2016)} \]

\[ \text{> 99\% globally optimal} \]

\[ \text{< 3 minutes on a laptop.} \]

\[ \text{Zhang & Lavaei, Math. Programming (2020)} \]
Linear Memory & Linear time

real-world

\[ n = 3,000 \]

RAM

\[ n \]

1 GB

Time

\[ n \]

1.7 min

previously unthinkable

\[ n = 300,000 \]

100x

100x

100x

100x

100 GB

27.8 hrs

1.7 min
Conclusions - Thank you!

1. Optimization is hard. Nonconvexity & extreme scale.
2. Engineers optimize w intuition. Underlying structure!
3. Formalize intuition. Scalable algorithms with strong guarantees. A qualitatively different perspective.
4. Next step: Solve real world problems!