Dynamic Sensitivity Functions and Professor Pai

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Trajectory Sensitivities

- How sensitive are the trajectories of the *nonlinear* power system to variations in system parameters?
- The concept: As the system becomes more stressed, the state trajectories will become more sensitive to parameter variations
- Thus the trajectory sensitivities can be used as a measure of system security

System Equations

- DE systems:
  - x = machine variables, α = system parameters

$$\dot{x} = f(x, \alpha) \ , \qquad x(t_o) = x_o$$

- DAE systems:
  - x = machine variables, y = load-flow and stator algebraic variables, α = system parameters

$$\dot{x} = f(x, y, \alpha), \qquad x(t_o) = x_o$$

$$0 = g(x, y, \alpha), \qquad y(t_o) = y_o$$

Sensitivity Functions

- Defining

$$x_\alpha = \frac{\partial x}{\partial \alpha} \equiv S$$

- And

$$A(t,\alpha) = \left.\frac{\partial f(t,x,\alpha)}{\partial x}\right|_{x=x(t,\alpha)}$$

$$B(t,\alpha) = \left.\frac{\partial f(t,x,\alpha)}{\partial \alpha}\right|_{x=x(t,\alpha)}$$

Sensitivity Functions

- Evaluating A and B at α_0:

$$\dot{S}(t) = A(t,\alpha_o)S(t) + B(t,\alpha_o)\,, \qquad S(t_o) = 0$$

- Then the new system is:

$$\dot{x} = f(t,x,\alpha)\,, \qquad x(t_o) = x_o$$

$$\dot{x}_\alpha = \left[\frac{\partial f}{\partial x}\right]x_\alpha + \left[\frac{\partial f}{\partial \alpha}\right], \quad x_\alpha(t_o) = 0$$

Initial Condition Sensitivity

- Defining

$$x_\beta = \frac{\partial x}{\partial \beta} \equiv S$$

- The system becomes:

$$\dot{x} = f(t, x), \qquad x(t_o) = \beta$$

$$\dot{x}_\beta = \left[\frac{\partial f}{\partial x}\right] x_\beta, \qquad x_\beta(t_o) = I$$

Single-machine example

- The classical swing equations for the single-machine system:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{M}(T_M - P_{m1}\sin x_1 + P_{m2}\sin 2x_1 - D_m x_2)$$

- The initial condition vector for this system is:

$$\beta = \begin{bmatrix} x_1(0) & x_2(0) \end{bmatrix}^T$$

Initial Condition Sensitivities

- Sensitivity Matrix

$$S = \begin{bmatrix} x_3 & x_5 \\ x_4 & x_6 \end{bmatrix} = \begin{bmatrix} \frac{\partial x_1}{\partial \beta_1} & \frac{\partial x_1}{\partial \beta_2} \\ \frac{\partial x_2}{\partial \beta_1} & \frac{\partial x_2}{\partial \beta_2} \end{bmatrix}$$

- Equilibrium point for T_M = 0.91
  - x_1 = 0.414
  - x_2 = 0
- Equilibrium point for T_M = 3.0
  - x_1 = 1.548
  - x_2 = 0

Peaks of Sensitivities

- $T_M = 0.91$

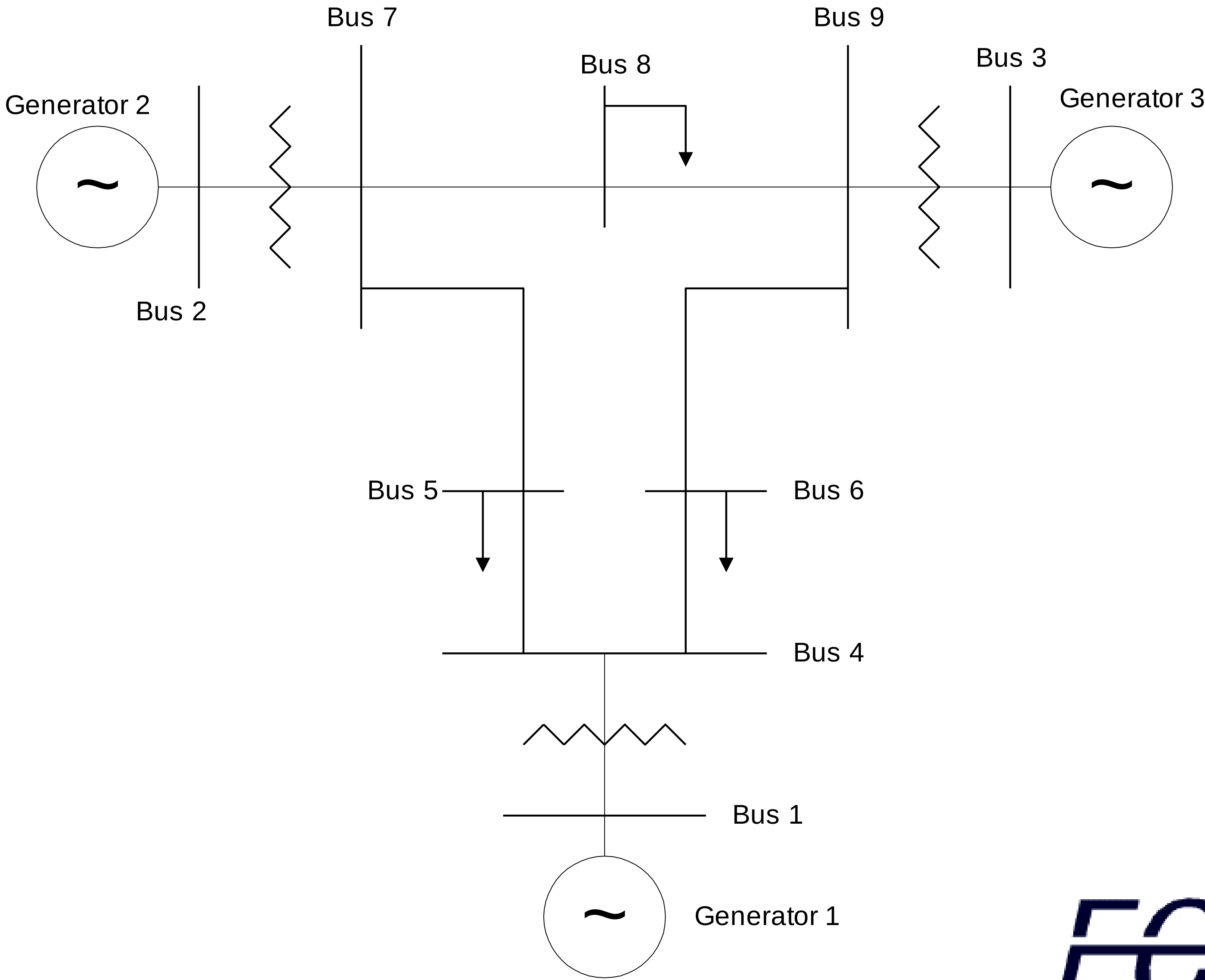
$x_1(0)$	$x_2(0)$	Peak of x_3	Peak of x_6
0.414	0	1	1
0.414	4	1	1
0.414	25	1	1.17
0.414	27	1.03	2.32
0.414	28	2.24	4.24
0.414	29	7.83	13.04

- $T_M = 3.0$

$x_1(0)$	$x_2(0)$	Peak of x_3	Peak of x_6
1.548	0	1	1
1.548	3	1	1
1.548	4	22.71	15.68

- System shows stress closer to the equilibrium point

Three-machine test system



Three-Machine system

- $n = 5$, $p = 3$, so $n + np = 20$ equations
- Initial conditions are no longer picked arbitrarily
  - Fault, with a specified clearing time, is applied to system
  - State variables at the clearing time are used as initial conditions for the sensitivity calculations
- Machines which are closer to losing synchronism will be more sensitive to parameter variations

Normalization of Parameter Sens.

- Absolute sensitivities do not give enough information
- ***Increase*** in sensitivity as clearing time increases indicates critical machines and contingencies
  - State at t_{cl} is used as the initial condition for the post-fault system
- Normalize with respect to peak sensitivities at a small clearing time
  - Can be calculated off-line and stored

Normalized Parameter Sens.

- Nominal Case

Clearing time	x_{13} (machine 2)	x_{20} (machine 3)
approx. 0 s	1	1
0.01 s	1.02	1.01
0.05 s	1.53	1.04
0.10 s	2.37	1.15
0.12 s	**2.78**	**1.22**
0.15 s	3.57	1.39
0.20 s	5.92	2.15
0.23 s	14.33	5.33
0.24 s	209.17	70.01
0.25 s	UNSTABLE	

Normalized Parameter Sens.

- Smaller inertias on machines 2 and 3

Clearing time	**x_{13} (machine 2)**	**x_{20} (machine 3)**
approx. 0 s	1	1
0.01 s	1.24	1.14
0.05 s	2.09	1.34
0.10 s	4.47	2.25
0.12 s	**11.00**	**5.42**
0.127 s	66.58	30.14
0.13 s	UNSTABLE	

Application to DAE Systems

- Define

$$w_1 = \frac{\partial x}{\partial \alpha}, \quad w_2 = \frac{\partial y}{\partial \alpha}$$

- Sensitivities are found using:

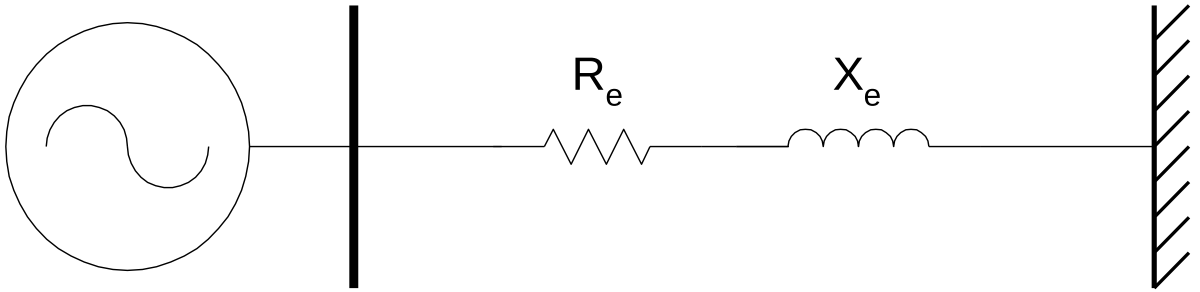
$$\dot{x} = f(x, y, \alpha), \qquad x(t_o) = x_o$$

$$0 = g(x, y, \alpha), \qquad y(t_o) = y_o$$

$$\dot{w}_1 = \frac{\partial f}{\partial x} w_1 + \frac{\partial f}{\partial y} w_2 + \frac{\partial f}{\partial \alpha}, \qquad w_1(0) = 0$$

$$0 = \frac{\partial g}{\partial x} w_1 + \frac{\partial g}{\partial y} w_2 + \frac{\partial g}{\partial \alpha}, \qquad w_2(0) = 0$$

Single-machine Infinite-bus



- Peak sensitivities

System	$X_e = 0.406$ $T_M = 0.976$	$X_e = 0.271$ $T_M = 1.469$
Peak of $\partial\delta/\partial T_M$	1.093	2.75
Peak of $\partial\delta/\partial X_e$	23.88	53.14

Implications

- The peak values of the sensitivities are good indicators about the proximity of the initial condition of the post-fault system to the stability boundary
- Indication about which machine is the most sensitive to a particular contingency
- Can rank contingencies according to their severity