Inverse Problems: Perspectives, Analysis and Insights (PAI)

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PaiFest!

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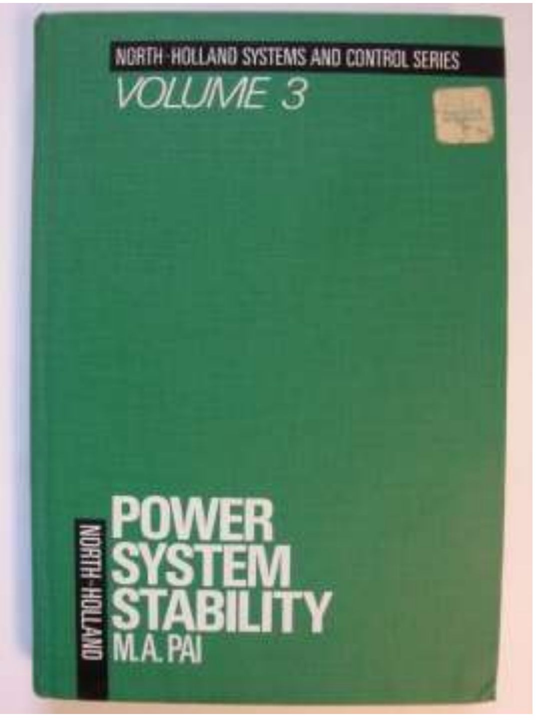
In the beginning…



⇒ "Energy Functions, Transient Stability and Voltage Behaviour in Power Systems"

Sabbatical 1997

- Stockholm or Champaign.

- Corn fields.
- Cross-country meets in unusual places.
  – "Chrisman Cow Chip Classic"
- Lost in East St. Louis.
- Driving in Chicago on 10-lane freeways.

- Trajectory sensitivities.
- Hybrid dynamical systems.
- Object-oriented modelling.

} Tools for solving problems in a different way.

- Inverse problems.

Inverse problems

- Forward problems => Given parameters determine the trajectory.
- Inverse problems => Given the desired trajectory determine parameters.

- Parameter estimation.
- Dynamic embedded optimization.
- Limit cycles.
- Grazing/reachability.
- Stability boundary.

Trajectory sensitivities

- Consider a trajectory (or flow) $x(t) = \phi(x_0, t)$ generated by simulation.
- Linearize the system around the *trajectory* rather than around the equilibrium point.

$$\begin{aligned}\Delta x(t) &= \frac{\partial \phi(x_0, t)}{\partial x_0} \Delta x_0 + \text{higher order terms} \\ &\approx \Phi(t) \Delta x_0\end{aligned}$$

- Trajectory sensitivities describe the change in the trajectory due to (small) changes in parameters and/or initial conditions.
- Provides gradient information for iteratively solving inverse problems.

Trajectory sensitivity evolution

Along smooth sections of the trajectory

System evolution

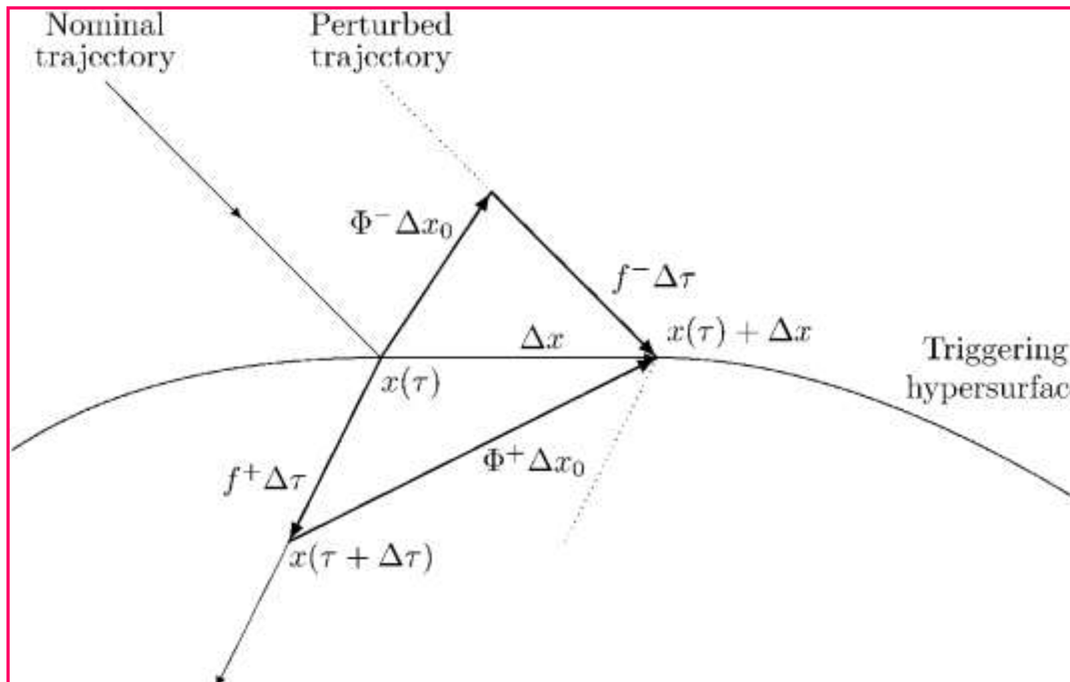
$$\dot{x} = f(x), \qquad x(0) = x_0$$

Sensitivity evolution

$$\dot{\Phi} = \left.\frac{\partial f}{\partial x}\right|_{x(t)} \Phi, \qquad \Phi(0) = I$$

At an event

$$\Phi(\tau^+) = \Phi(\tau^-) - (f^+ - f^-)\,\frac{\partial \tau}{\partial x_0}$$



Trajectory sensitivity computation

Implicit numerical integration allows efficient computation of trajectory sensitivities.

System evolution

$$\dot{x} = f(x)$$

Trapezoidal integration

$$x^{k+1} = x^k + \tfrac{h}{2}\Big(f(x^k) + f(x^{k+1})\Big)$$

Each integration timestep involves a Newton solution process.

- The Jacobian $\underbrace{\left(\frac{h}{2} Df - I\right)}$ must be formed and factored.

Sensitivity evolution

$$\dot{\Phi} = Df(x(t))\Phi$$

Trapezoidal integration

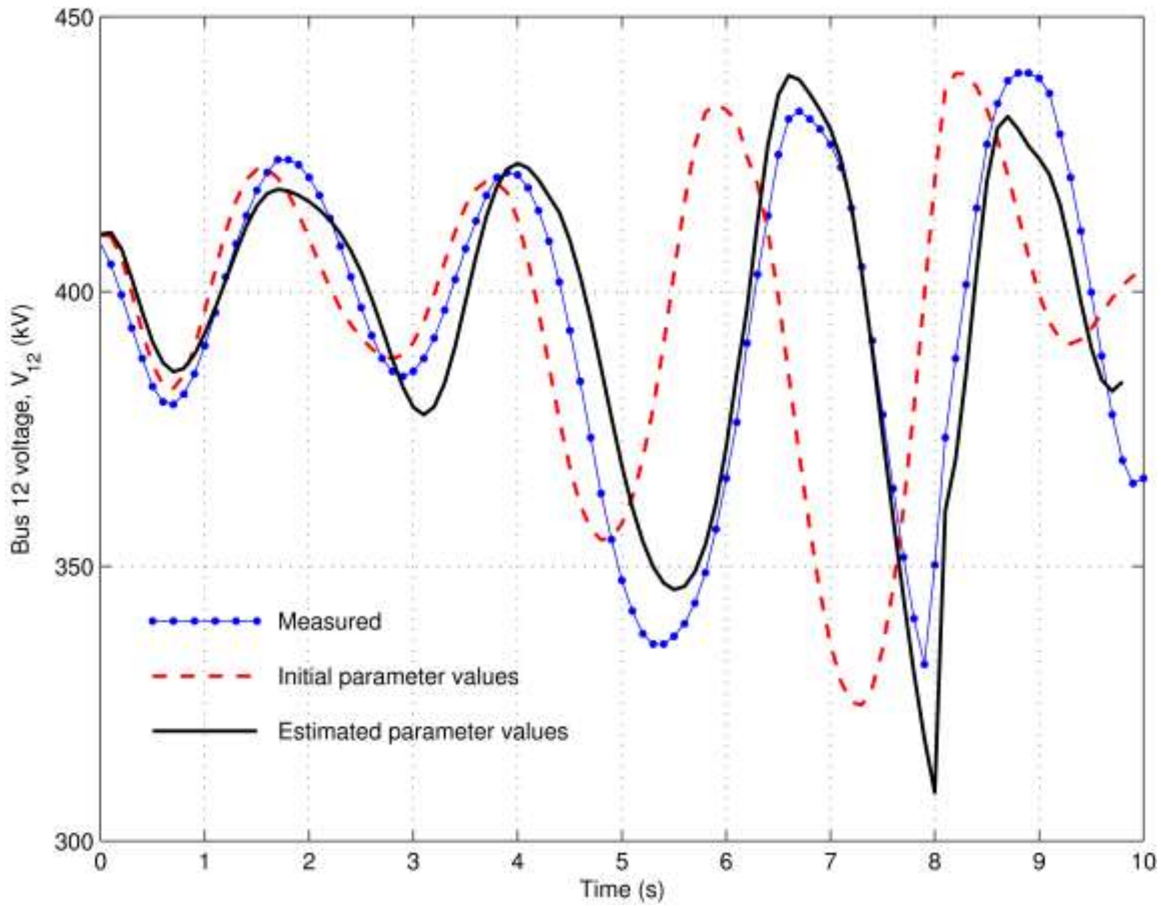
$$\Phi^{k+1} = \Phi^k + \tfrac{h}{2}\Big(Df(x^k)\Phi^k + Df(x^{k+1})\Phi^{k+1}\Big)$$

$$\Rightarrow \underbrace{\left(\frac{h}{2} Df(x^{k+1}) - I\right)}\Phi^{k+1} = -\Big(\tfrac{h}{2} Df(x^k) + I\Big)\Phi^k$$

Already factored

Parameter estimation

- Determine *well conditioned* parameters.
- Estimate those parameters.
- Nonlinear least-squares problem.

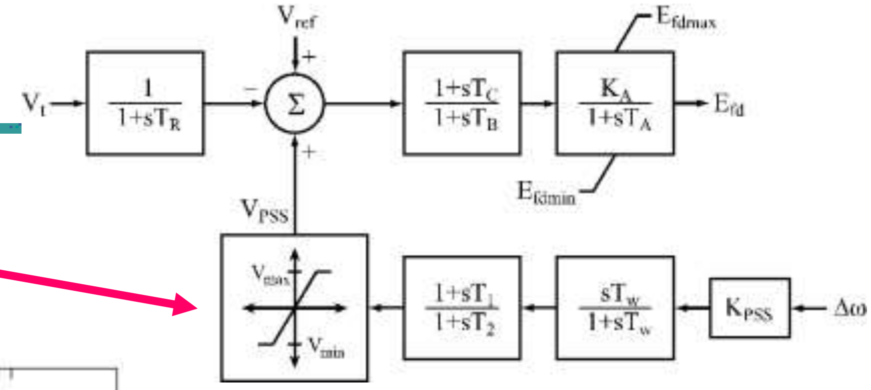


Real world example:
Disturbance on the 330kV Scandinavian network.

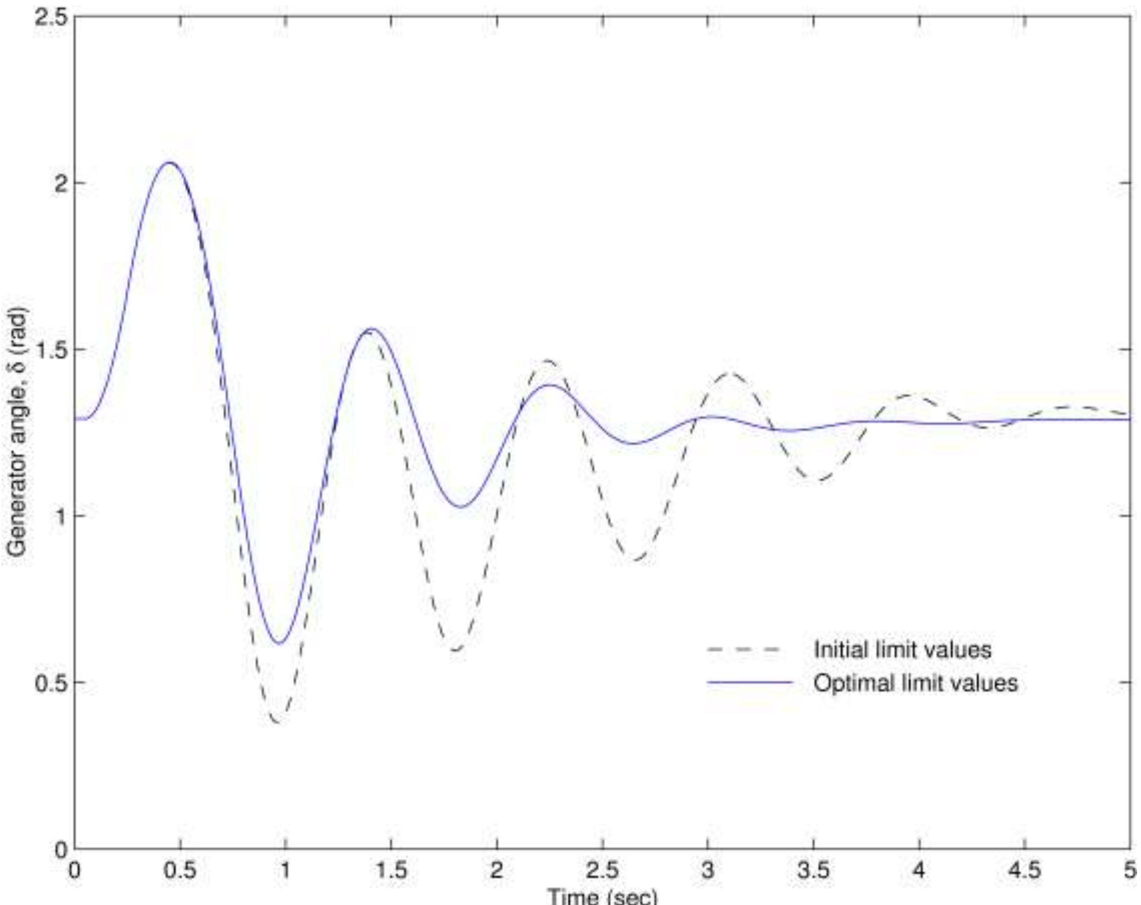
A voltage measurement was used to estimate various parameters, including the switching time of a transmission line.

Optimization example

Improve damping by optimizing PSS limits.



Generator AVR/PSS



Optimization adjusted lower PSS limit from -0.1 to -0.33.

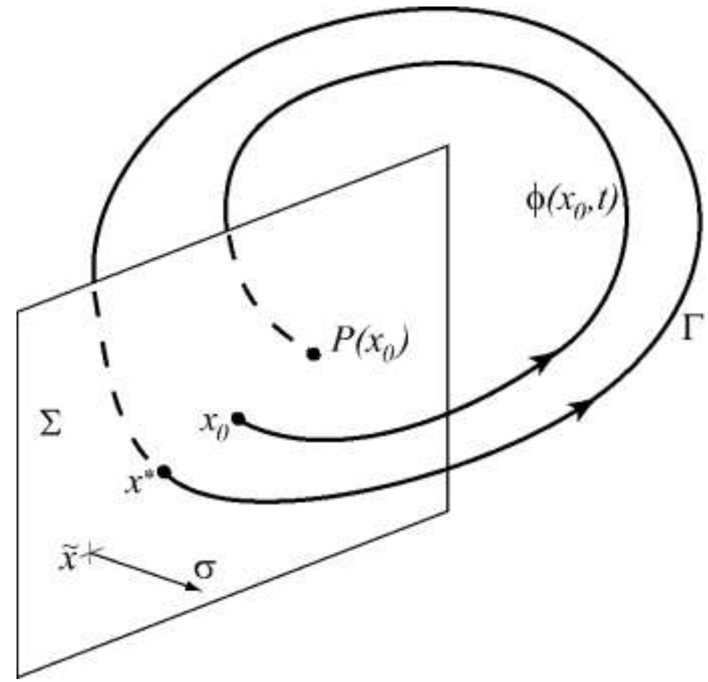
Limit cycles (periodic behaviour)

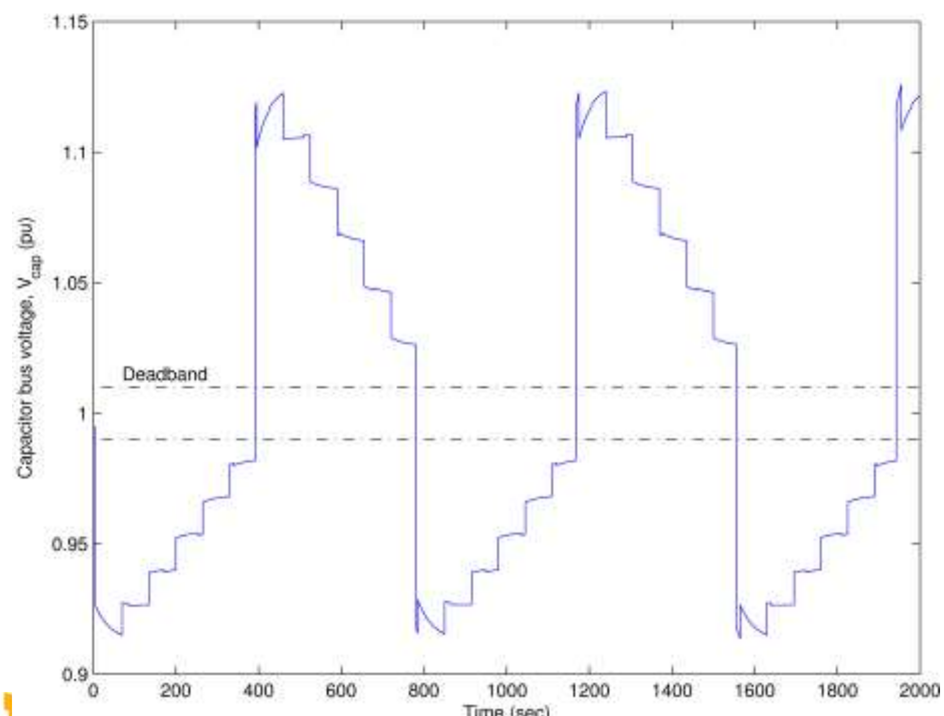
Analysis and computation are based on Poincaré map concepts.

Solve $$F_l(x) := \phi(x, \tau_r(x)) - x = 0$$

where $\tau_r(x)$ is the return time.

- Reliable convergence, even for unstable limit cycles.





Limit cycles may be non-smooth.

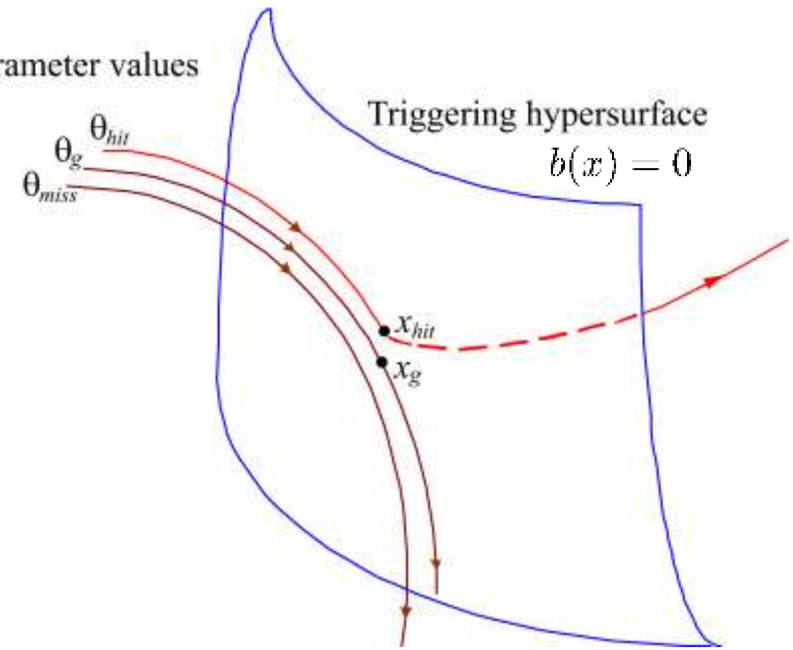
Example: Interactions between a tap-changing transformer and a switched capacitor.

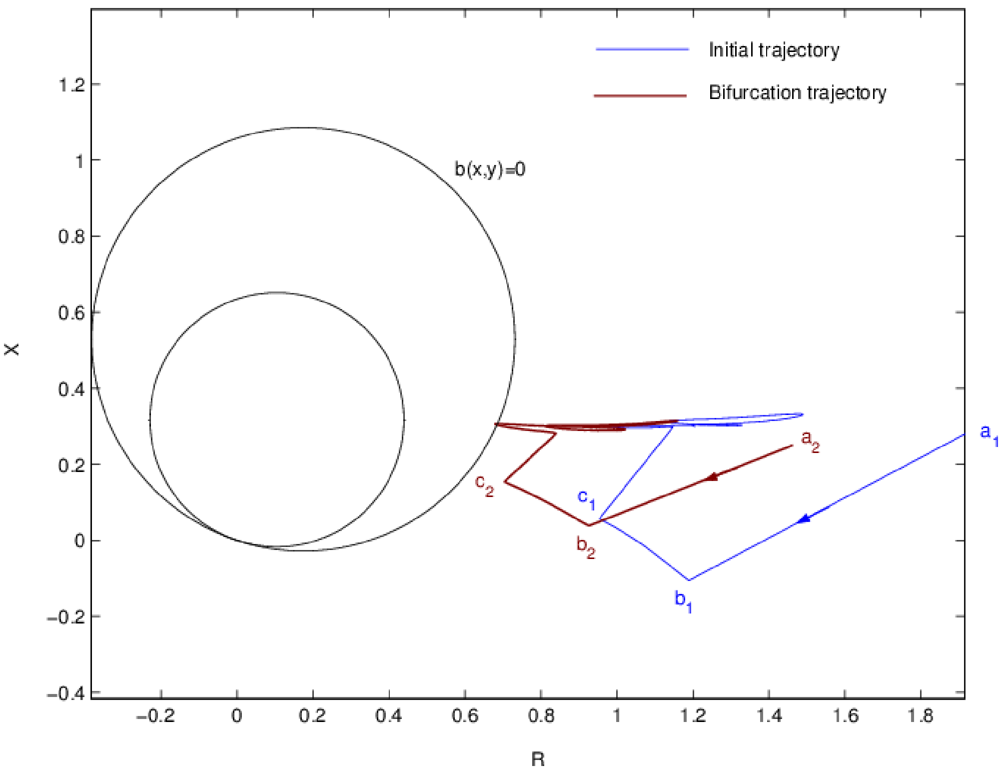
Grazing phenomena

Tangential encounter between the trajectory and a specified surface.

Solve

$$\phi(x_0(\theta_g), t_g) - x_g = 0$$
$$b(x_g) = 0$$
$$\nabla b(x_g)^\top f(x_g) = 0$$





Example:
Distance protection.

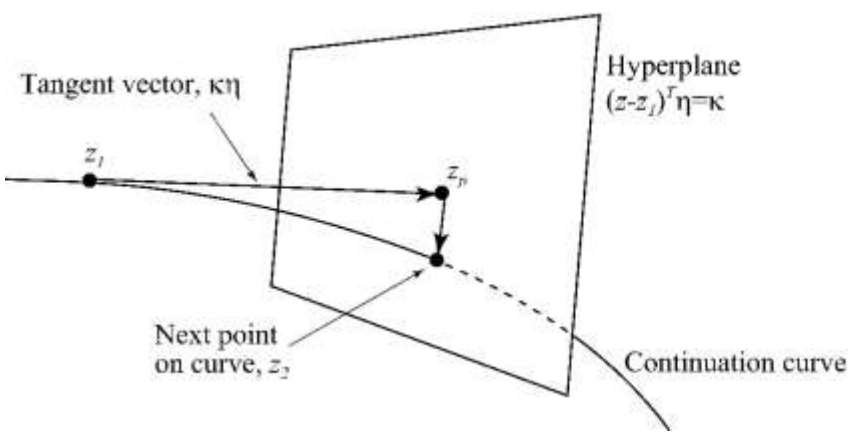
Shooting methods

Point solutions: Solve $F(z) = 0$ where F incorporates the flow ϕ.

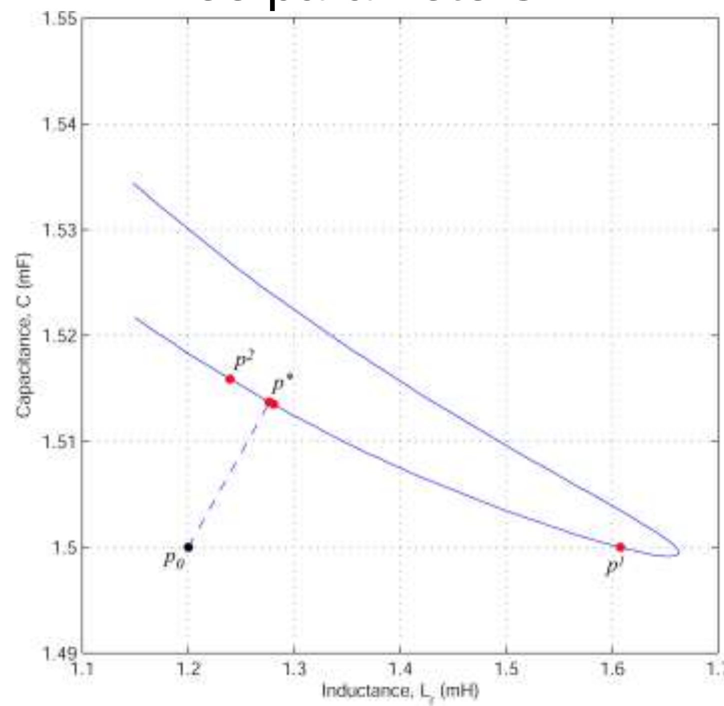
- Newton solution: $z^{k+1} = z^k - DF(z^k)^{-1}F(z^k)$
- Evaluation of F requires simulation to determine ϕ.
- Evaluation of DF requires trajectory sensitivities Φ.

Continuation process: One free parameter => under-determined system $F: \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$

Solutions of $F(z) = 0$ describe a 1-manifold.

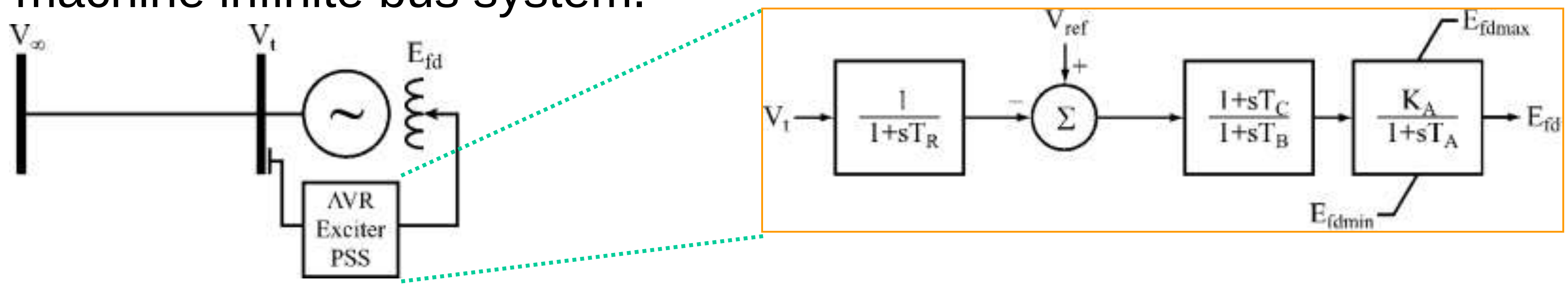


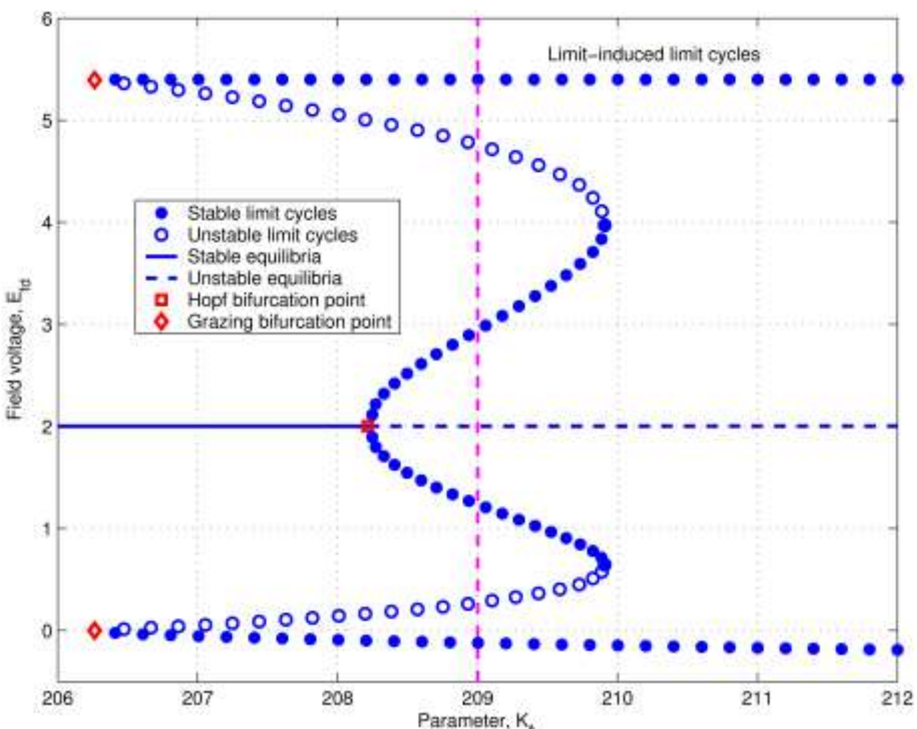
Optimization: Multiple free parameters.

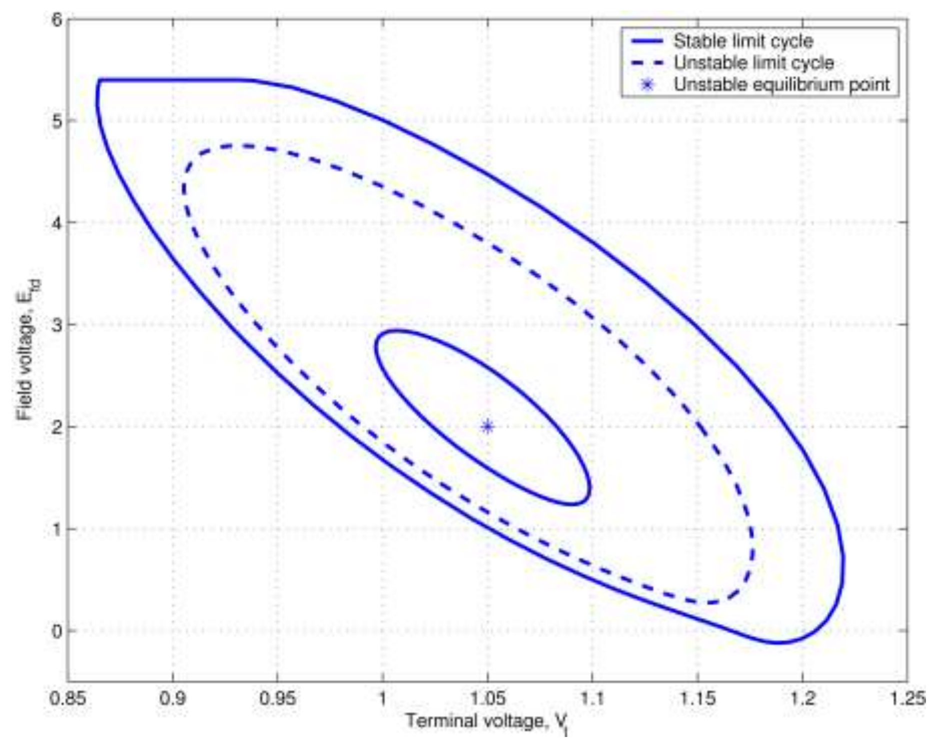


Example: bifurcations

Single machine infinite bus system.



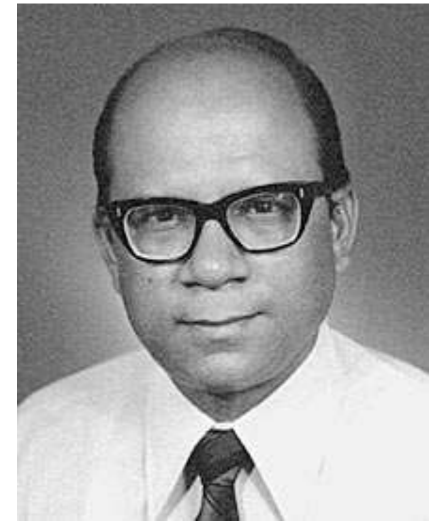




"I'd like to be" by C.J. Dennis

I'd like to be a pie-man,
and ring a little bell,
Calling out "hot pies, hot pies to sell".
Apple pies and meat pies,
Cherry pies as well.
Lots and lots and lots of pies
More than you can tell.
Big rich pork pies,
Oh the lovely smell!

But I wouldn't be a pie-man if.....
I wasn't very well!



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