A Unified Framework for Birthday Celebration : Happy Birthday Professor Gross

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***Abstract*—In this paper, we investigate the impacts of celebrating your advisor's birthday. The birthday cake is a prime mover for celebrating a birthday. We illustrate how tempting a birthday cake can be, and also how the cake can be used to drive in key concepts in research. We propose an algorithm on "How to cut the cake". We conclude with some examples of what we have learned from Prof. Gross.**

I. The algorithm

- Step 1. Get a cake
- Step 2. Blow out the candles
- Step 3. Make a wish
- Step 4. Cut the cake
- Step 5. Let your students eat the cake

II. The Cake

Fig. 1. Electricity Cake

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We know that,

$$\mathcal{F}_i = \sum_{j\neq i} \mathcal{F}_{ij} = \sum_{j\neq i} \lambda_{ij} p_j \tag{1}$$

In long term we also know that,

$$\sum_{j\neq i} \lambda_{ij} p_j = \lambda_i p_i \tag{2}$$

In a homogeneous Markov chain holds that,

$$\lambda_i = \sum_{j\neq i} \lambda_{ji} \tag{3}$$

Then, from (2) and (3) we obtain,

$$\mathcal{F}_i = \lambda_i p_i = p_i(\sum_{j\neq i} \lambda_{ji}) \tag{4}$$

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In order to show that $A = LU$, we need to compute the element i, j of LU that we are going to call B_{ij}.

$$[LU]_{ij} = \sum_{m=1}^{N} l_{im} u_{mj} = B_{ij} \tag{5}$$

Using the definitions for L and U and the fact that L is lower triangular and U upper triangular, we obtain,

$$B_{ij} = \sum_{m=1}^{\min(i,j)} a_{im}^{(m-1)} u_{mj} \tag{6}$$

- Case $i \geq j$
  In this case $\min(i, j) = j$, hence:

$$B_{ij} = \sum_{m=1}^{j} a_{im}^{(m-1)} u_{mj} \tag{7}$$

$$= \sum_{m=1}^{j-1} a_{im}^{(m-1)} u_{mj} + a_{ij}^{(j-1)} u_{jj} \tag{8}$$

$$= \sum_{m=1}^{j-1} a_{im}^{(m-1)} u_{mj} + a_{ij}^{(j-1)} \tag{9}$$

$$= \sum_{m=1}^{j-1} a_{im}^{(m-1)} a_{mj} + a_{ij}^{(j-1)} \tag{10}$$

Using the fact that in the last expression,

$$a_{km}^{(m)} = a_{kj}^{(m-1)} - a_{km}^{(m-1)} a_{mj}^{(m)} \tag{11}$$

We obtain,

$$B_{ij} = \sum_{m=1}^{(j-1)} (a_{ij}^{(m-1)} - a_{ij}^{(m)}) + a_{ij}^{(j-1)} \tag{12}$$

$$= a_{ij}^{(0)} - a_{ij}^{(j-1)} + a_{ij}^{(j-1)} = a_{ij}^{(0)} = a_{ij} \tag{13}$$

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- Case $i < j$ In this case $\min(i, j) = i$, hence:

$$B_{ij} = \sum_{m=1}^{i} a_{im}^{(m-1)} u_{mj} = \sum_{m=1}^{i-1} a_{im}^{(m-1)} u_{mj} + a_{ii}^{(m-1)} u_{ij} \tag{14}$$

$$= \sum_{m=1}^{i-1} a_i m^{(m-1)} a_{mj}^{(m)} + a_{ii}^{(i-1)} a_{ij}^{(i)} \tag{15}$$

Using the normalization equation ,

$$a_{ii}^{(i-1)} a_{ij}^{(i)} = a_{ij}^{(i-1)} \tag{16}$$

and (11), in the same way that in the previous case, we have that,

$$B_{ij} = \sum_{m=1}^{(i-1)} (a_{ij}^{(m-1)} - a_{ij}^{(m)}) + a_{ij}^{(i-1)} = \tag{17}$$

$$a_{ij}^{(0)} - a_{ij}^{(i-1)} + a_{ij}^{(i-1)} = a_{ij}^{(0)} = a_{ij} \tag{18}$$