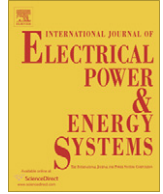




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## Congestion rents and FTR evaluations in mixed-pool-bilateral systems

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### ABSTRACT

Evaluation of key metrics for congestion management, including the congestion rents and the financial transmission rights (FTR) payoffs, requires the efficient allocation of transmission services and the calculation of appropriate locational marginal prices (LMPs). This requirement is particularly acute when there are bilateral transactions coexisting with the centralized pool markets. We propose a new formulation for this purpose, which captures explicitly the contribution of the bilateral transactions to the social welfare. The proposed formulation effectively integrates the pool market and the bilateral transactions on a consistent basis and results in the more efficient allocation of the transmission resources than the conventional tool. We assess analytically the capabilities of the proposed formulation and solution and quantify the improvements in the evaluations over those done using the conventional approach. Such improvements are also illustrated using simulation results on a wide range of test systems including the IEEE 118-bus network.

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### 1. Introduction

The advent of open access transmission and the spread of competitive markets [1] in electricity have resulted in the growing prominence of transmission congestion. Congestion occurs whenever the provision of transmission services required by the preferred generation/demand schedule exceeds the physical capability of the grid. Congestion introduces unavoidable losses in efficiency and so the benefits foreseen through restructuring may not be fully realized. There is a growing awareness that congestion is a major obstacle to vibrant competitive electricity markets [2]. Therefore, effective management of congestion is a critically important contributor to the smooth functioning of competitive electricity markets [3].

Among the various schemes proposed to manage congestion, the market-based mechanisms using the locational marginal prices (LMPs) [2,4] have become widely used. Central to these schemes is the centralized day-ahead pool markets run by the independent entity which we call the *independent grid operator* or IGO. We use the generic IGO term to refer to the organization responsible for market operations and for operating and controlling the transmission network. The IGO term is general and includes entities such as ISOs, RTOs and TSOs. The pool players sell [buy] energy directly to [from] the IGO through the sealed offers [bids] submitted. Each offer [bid] specifies the MWh amounts of power the player is willing to sell [buy] and the per MWh minimum [maximum] price it is

willing to accept [pay]. From the bids and offers submitted, the IGO determines the market outcomes by maximizing the social welfare [5]. In this way, the LMP at each network node is determined. In a lossless system, the presence of congestion is signaled by nonzero LMP differences between node pairs. The IGO provides the transmission services necessary to deliver the energy traded in these markets and also collects transmission usage charges arising from the LMP differences. The sum of these usage charges is referred to as the *merchandising surplus* [6,7]. Coexisting with the centralized markets are the *bilateral transactions* for direct energy trading. Each transaction involves a selling entity and a buying entity. Whenever these two entities in a bilateral transaction are located at different nodes, the consummation of the transaction requires transmission services. One of the two entities is responsible for procuring from the IGO the required transmission services for the particular transaction and for paying to the IGO the *congestion charges* [4] determined by the product of the LMP difference and the MW amount of the transaction. We use the term *congestion rents* to refer to the sum of the congestion charges collected from the bilateral transactions and the merchandising surplus from the pool players due to congestion [8]. For this paper, we restrict ourselves to the consideration of congestion in the day-ahead hourly markets operated by the IGO.

The inherent volatility of electricity markets introduces uncertainty in the LMPs and consequently, in the congestion rents. In order to protect the transmission customers from the impact of such uncertainty, financial tools such as financial transmission rights (FTR) [4,6,9] are introduced. The FTR are issued by the IGO in a specified amount of MW for a pair of given *from* and *to* nodes

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and with a specified duration. The FTR holder is reimbursed a share of the congestion rents collected by the IGO in the amount equal to the product of the specified MW quantity and the LMP difference between the two specified nodes for each hour of the specified duration [4]. A transmission customer holding appropriate FTR is provided price certainty in light of the reimbursement by the IGO of the congestion charges for the MW amount of the FTR held.

The evaluation of the congestion rents and the payoffs to the FTR holders is critical for the effective deployment of the LMP-based congestion management scheme. The key challenges are the efficient allocation of the limited transmission services, the appropriate representation of the social welfare and the evaluation of the appropriate LMP signals. The determination of these quantities entails solving the so-called *transmission scheduling problem* or TSP [10–15]. While the TSP for markets without considering bilateral transactions has been investigated [10–13], the consideration of markets incorporating both the pool and the bilateral transaction paradigms has received far less attention. The conventional approach consists of a separate decision-making process for the bilateral transactions and the pool players. Bilateral transactions request transmission services from the IGO ahead of the clearing of the day-ahead market. The provision of transmission services depends on the *available transfer capability* [16] for the requested *from/to* node pair and is determined on a first-come-first-served basis [17]. Then, in the day-ahead market, the transactions that have obtained the transmission services are represented as fixed injections and withdrawals. The pool market is cleared using a modified version of the TSP formulation [13,14,18]. This version maximizes the social welfare subject to the *remaining* transfer capability of the network, evaluated with these fixed injections and withdrawals taken into account. The solution of the TSP provides the market clearing quantities for each pool player and the LMPs at each node. The bilateral transaction congestion charges are determined based on these LMPs. An implicit assumption in this approach is the very high willingness to pay for the congestion charges on the part of the bilateral transactions.

Such a formulation is workable for the evaluation of congestion rents and FTR payoffs as long as the fraction of the total load served by the bilateral transactions is small. In practical situations, however, this condition may not hold [19] and, consequently, the use of the TSP formulation becomes questionable. The two key issues of particular concern are:

- the separate decision-making process for the bilateral transactions and the pool players; and
- the complete disregard of the willingness to pay for the congestion charges by the bilateral transactions since they are treated as price takers.

These concerns become particularly acute as the fraction of total load served by bilateral transactions increases. This paper aims to effectively address these concerns by proposing a new framework in which the bilateral transactions can specify their willingness-to-pay for the congestion charges and be provided transmission services accordingly. We develop a general formulation that allows the joint scheduling by the IGO of transmission services to both the pool players and the bilateral transactions. The objective is the optimization of social welfare with the contribution of the bilateral transactions explicitly represented. We refer to the proposed formulation as the *transmission allocation problem* or TAP. In this formulation, we consider the transmission services for the bilateral transactions as decision variables whose optimal values are determined *simultaneously* with the outcomes of the pool market. This approach is similar to the notion used in Ref. [20] to determine the FTR and the energy sale quantities in a combined FTR and energy auction. The proposed TAP solution provides

transmission service to the bilateral transactions in accordance with their willingness to pay and, as such, on a basis consistent with the service provision to the pool players. Moreover, when compared to the solutions of the conventional TSP, the market outcomes obtained using the TAP tool have larger social welfare in the presence of congestion. Also, the proposed TAP allows the evaluation of LMPs that portray the integrated nature of the pool and bilateral transaction markets. These LMPs serve in the computation of the congestion rents and the payoffs of the FTR. These aspects are discussed in detail and illustrated with numerical results for various systems.

This paper contains five additional sections. We devote Section 2 to constructing the proposed TAP formulation. In Section 3, we analyze the structural characteristics of the TAP formulation and its solution, with particular emphasis on the pricing information. In Section 4, we study the role of the TAP solution in the evaluation of the congestion rents and the FTR payoffs. We demonstrate in Section 5 the effectiveness of the TAP in the evaluation of the congestion rents and FTR payoffs using a wide range of test systems. We discuss future extensions of the paper in the concluding section.

## 2. Construction of the formulation

We consider a power network consisting of  $N + 1$  buses and  $L$  lines. We focus on the single hour  $h$  and obtain a snapshot of the system. We denote the set of buses by  $\mathcal{N} \triangleq \{0, 1, \dots, N\}$ , with bus 0 being the slack bus. We denote the set of transmission lines and transformers that connect the buses in the set  $N$  by  $L \triangleq \{\ell_1, \ell_2, \dots, \ell_L\}$ . We associate with each element  $\ell \in L$  the ordered pair  $\ell = (i, j)$  for the line  $\ell$ , whose series admittance is  $g_\ell - jb_\ell$ . Under our convention, the direction of the real power flow  $f_\ell$  in line  $\ell$  is from node  $i$  to node  $j$  so that  $f_\ell \geq 0$ . We construct  $\mathbf{f} \triangleq [f_1, f_2, \dots, f_L]^T$ . We define  $\mathbf{B}_d \triangleq \text{diag}\{b_1, b_2, \dots, b_L\}$  to be the diagonal  $L \times L$  branch susceptance matrix and  $\mathbf{B} \triangleq \mathbf{A}^T \mathbf{B}_d \mathbf{A}$  to be the reduced nodal susceptance matrix. We denote by  $\mathbf{A}$  the reduced incidence matrix and by  $\mathbf{b}_0$  the column of the nodal susceptance matrix corresponding to the slack bus 0 [21].

We assume, without loss of generality, that at each network node  $n = 0, 1, \dots, N$ , there is a single pool seller and a single pool buyer.<sup>1</sup> We denote by  $\bar{p}_n^s[\bar{p}_n^b]$  the maximum amount of power the seller [buyer] at node  $n$  is willing to sell [buy]. The submitted marginal seller offer [buyer bid] is integrated and we denote this integral by  $\beta_n^s(p_n^s)[\beta_n^b(p_n^b)]$ . We assume  $\beta_n^s(\cdot)[\beta_n^b(\cdot)]$  to be nonnegative, differentiable and convex [concave] functions. We define  $\mathbf{p}^s \triangleq [p_1^s, p_2^s, \dots, p_N^s]^T$  and  $\mathbf{p}^b \triangleq [p_1^b, p_2^b, \dots, p_N^b]^T$ . We represent each bilateral transaction  $\bar{\omega}^w$  with the receipt point (*from* node)  $m^w$ , the delivery point (*to* node)  $n^w$  and the *desired* transaction amount  $\bar{t}^w$  MW by the ordered triplet  $\bar{\omega}^w \triangleq \{m^w, n^w, \bar{t}^w\}$ . The set  $W \triangleq \{\bar{\omega}^1, \bar{\omega}^2, \dots, \bar{\omega}^W\}$  denotes the set of desired bilateral transactions in hour  $h$ . We define  $\bar{\mathbf{t}} \triangleq [\bar{t}^1, \bar{t}^2, \dots, \bar{t}^W]^T$  to be the vector of desired transmission amounts. Each bilateral transaction  $\bar{\omega}^w$  submits to the IGO its request for the required transmission services for its desired transaction.

We first consider the objective function in the formulation. In the conventional TSP formulation, the bilateral transactions are considered as non-dispatchable and their contributions to the social welfare are not considered. As such, the *social welfare* in the TSP contains the contributions of only the pool players:

<sup>1</sup> In the case of multiple offers and multiple bids at a single node, we adopt the following scheme. The offers of multiple sellers at a node are combined into a composite offer function submitted by a single seller. Similarly, the bids of multiple buyers at a node are combined into a composite bid function submitted by a single buyer.

$$\mathcal{F}(p_0^s, p_0^b, \mathbf{p}^s, \mathbf{p}^b) = \sum_{n=0}^N [\beta_n^b(p_n^b) - \beta_n^s(p_n^s)]. \quad (1)$$

However, in addressing the scheduling of transmission services to both the pool players and the bilateral transactions on a consistent basis, we need to include an additional term, which expresses the contribution of each bilateral transaction to the social welfare. We digress briefly to formulate this term.

We consider the bilateral transaction  $\bar{\omega}^w \triangleq \{m^w, n^w, \bar{t}^w\}$  with the seller at node  $m^w$  and the buyer at node  $n^w$ . We denote by  $\mathcal{C}(t^w)$  the seller's costs as a function of the transmitted MW amount  $t^w$  of the transaction,  $0 \leq t^w \leq \bar{t}^w$ . We use  $\mathcal{B}(t^w)$  to represent the buyer's benefits. The transmission services provided to deliver  $t^w$  MW for the transaction  $\bar{\omega}^w$  then contribute  $\mathcal{B}(t^w) - \mathcal{C}(t^w)$  to the social welfare, i.e., the difference between the consumption benefits and the production costs of the transaction. However, since the actual costs and benefits information is confidential and known only to the bilateral transaction parties, this difference cannot be readily computed. Rather, we assume that the bilateral transaction  $\bar{\omega}^w$  provides a function  $\alpha^w(\cdot) : [0, \bar{t}^w] \rightarrow [0, \infty)$  together with its transmission service request to indicate the maximum congestion charges it is willing to bear<sup>2</sup> for receiving the requested transmission services as a function of the transaction amount  $t^w$  delivered. We assume  $\alpha^w(\cdot)$  to be nonnegative, concave and differentiable.<sup>3</sup> A special case is the specification of a fixed per MW price  $\bar{\alpha}^w$ :

$$\alpha^w(t^w) = \bar{\alpha}^w t^w, \quad t^w \in [0, \bar{t}^w] \quad (2)$$

so that  $\bar{\alpha}^w$  acts as a price cap on the transmission price for transaction  $\bar{\omega}^w$ . In particular, when the customer has an un-limited willingness to pay,  $\bar{\alpha}^w$  becomes a large number  $\bar{\alpha}$

$$\alpha^w(t^w) = \bar{\alpha} t^w. \quad (3)$$

We can prove that if the LMP difference between the node pair  $n^w$  and  $m^w$  is independent of the willingness to pay of the transaction  $\bar{\omega}^w$ , as expressed by  $\alpha^w(t^w)$ , then, a rational decision maker responsible for requesting transmission service for the transaction  $\bar{\omega}^w$  maximizes its net profits by setting

$$\alpha^w(t^w) = B(t^w) - C(t^w), \quad \forall t^w \in [0, \bar{t}^w]. \quad (4)$$

The proof is given in [22]. The conditions in this statement hold, typically, when the market is competitive and the desired transaction amount  $\bar{t}^w$  of each transaction is small compared to the total system loads. These conditions do not contradict the assumption that the bilateral transactions serve a significant portion of the total loads, as their number may be large. The result in (4) allows us to modify the definition of the social welfare so as to explicitly incorporate the contributions of the bilateral transactions. We define the social welfare  $\mathcal{S}$  to be a measure of the net benefits of both the consummated bilateral transactions and the sales and purchases in the pool market:

$$\mathcal{S}(p_0^s, p_0^b, \mathbf{p}^s, \mathbf{p}^b, \mathbf{t}) \triangleq \sum_{n=0}^N [\beta_n^b(p_n^b) - \beta_n^s(p_n^s)] + \sum_{w=1}^W \alpha^w(t^w). \quad (5)$$

For the pool market only, with no bilateral transactions, (5) reduces to the expression for  $\mathcal{F}(\cdot)$  in (1).

Next, we consider the statement of the constraints in the provision of transmission services by the IGO. We first include in the nodal power flow balance equations the impacts of the bilateral transactions. For simplicity, we use the lossless DC model to represent the power flows in the network. The nodal impacts of the  $W$  bilateral transactions in the set  $\mathcal{W}$  are

$$p_n^w \triangleq \sum_{\substack{w=1 \\ \ni m^w=n}}^W t^w - \sum_{\substack{w=1 \\ \ni n^w=n}}^W t^w, \quad n = 0, 1, \dots, N. \quad (6)$$

We construct the  $N$ -dimensional vector

$$\mathbf{p}^w \triangleq [p_1^w, p_2^w, \dots, p_N^w]^T \quad (7)$$

and express the DC power flow equations as

$$\begin{cases} p_0^s - p_0^b + p_0^w = \mathbf{b}_0^T \boldsymbol{\theta}, \\ \mathbf{p}^s - \mathbf{p}^b + \mathbf{p}^w = \mathbf{B}\boldsymbol{\theta}, \end{cases} \quad (8)$$

where  $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_N]^T$  is the vector of nodal voltage angles. To ensure reliable system operations, the various physical and system operational constraints are considered. For simplicity, we represent all the transmission constraints by the real power line flow limits. To focus on the key characteristics of the formulation, we restrict our attention to only the base case conditions. All other constraints under both the base case and the contingencies can be represented in a similar way. Under these assumptions, the constraints [23] are stated as:

$$\mathbf{B}_b \mathbf{A} \boldsymbol{\theta} \leq \mathbf{f}^{\max}. \quad (9)$$

Thus, the IGO decision-making problem for the transmission service provision to both the bilateral transactions and the pool players is stated as:

$$\text{TAP} \begin{cases} \max & \mathcal{S}(p_0^s, p_0^b, \mathbf{p}^s, \mathbf{p}^b, \mathbf{t}) = \sum_{n=0}^N [\beta_n^b(p_n^b) - \beta_n^s(p_n^s)] + \sum_{w=1}^W \alpha^w(t^w) \\ \text{s.t.} & p_0^s - p_0^b + p_0^w = \mathbf{b}_0^T \boldsymbol{\theta} \leftrightarrow \mu_0, \\ & \mathbf{p}^s - \mathbf{p}^b + \mathbf{p}^w = \mathbf{B}\boldsymbol{\theta} \leftrightarrow \boldsymbol{\lambda}, \\ & \mathbf{B}_b \mathbf{A} \boldsymbol{\theta} \leq \mathbf{f}^{\max} \leftrightarrow \boldsymbol{\lambda}, \\ & \mathbf{0} \leq \mathbf{t} \leq \bar{\mathbf{t}} \leftrightarrow \boldsymbol{\eta}. \end{cases} \quad (10)$$

We refer to the general formulation in (10) as the TAP statement. For completeness, the dual variables of the various constraints are explicitly indicated.

The TAP decision variables are the sales/purchases amounts of the pool players and the amounts of transmission services provided to the bilateral transactions. The TAP solution determines *simultaneously* the optimal values taken by these variables to maximize the overall social welfare. As such, the bilateral transactions are not passive price takers, as in the conventional TSP, but active market participants competing on the same footing with the pool players for the IGO's limited transmission capabilities. In this way, the TAP optimization allocates the limited transmission services to customers – be they pool players or transactions – who value them most. In addition, the constraint dual variables provide very useful economic information. Note that these dual variables are determined under the explicit representation of the bilateral transactions. The ability to model the willingness to pay of each bilateral transaction with that of each pool buyer provides a realistic basis for the computation of these important economic signals. This is in direct contrast to the TSP formulation in which the only representation of the transactions is through their impacts on the real power line flow limits. We next examine the analytic properties of the TAP optimum.

<sup>2</sup> In practice, each bilateral transaction must pay an annual access fee for the connection to the network. In addition, the bilateral transaction pays transmission usage fees, consisting of the congestion charges and a contribution to the recovery of losses in the network. Our focus in this paper is purely on the congestion charges.

<sup>3</sup> For completeness, special bilateral transactions such as those requiring full acceptance or rejection may also be represented using discrete willingness to pay functions. In that case, the TAP becomes a mixed integer programming problem, whose solution is complicated and beyond the scope of this paper. Hence, we exclude the consideration of such special cases in this paper.

3. Analysis of the TAP solution

Our underlying assumption is that the TAP optimal solution  $(p_0^s, p_0^b, \mathbf{p}^s, \mathbf{p}^b, \mathbf{t}^*)$  exists. The solution specifies the pool market outcomes and the amount of transmission service provided to each bilateral transaction in  $\mathcal{W}$ . The pool seller at node  $n$  sells  $p_n^s$  MWh to the IGO in the specified hour  $h$ . Similarly, the pool buyer at node  $n$  purchases  $p_n^b$  MWh. The IGO allocates to the transaction  $\bar{\omega}^w = \{m^w, n^w, \bar{t}^w\}$  the amount of transmission services  $t^{w*}$ . The consummated transaction is  $\omega^w = \{m^w, n^w, t^{w*}\}$ . For  $t^{w*} = \bar{t}^w$ , the entire desired transaction amount  $\bar{\omega}^w$  takes effect; but for  $t^{w*} < \bar{t}^w$ , only a fraction of the desired transaction is undertaken.

The optimal values  $\mu_n^*$ ,  $n = 0, 1, \dots, N$ , of the dual variables associated with the nodal real power balance constraints determine the LMPs. The pool buyer at node  $n$  pays to the IGO the LMP  $\mu_n^*$  for each MWh bought in the pool while the seller at node  $n$  receives payments of  $\mu_n^* p_n^s$  from the IGO. Without congestion in the network, the LMPs at all the (lossless) network nodes are equal. When congestion occurs, however, the LMPs at each node may become different, indicating the different costs to supply an additional unit of energy at each node.

The optimal value  $\lambda_\ell^* \geq 0$  of the dual variable associated with the real power flow limit of line  $\ell$  measures the sensitivity of the social welfare with respect to the limiting capacity  $f_\ell^{max}$  of the line.  $\lambda_\ell^* > 0$  implies that line  $\ell$  constraint is binding and that line  $\ell$  is congested. Consequently, the per MW flow congestion charges for the usage of line  $\ell$  are  $\lambda_\ell^*$ . In terms of the power transfer distribution factor  $\phi_\ell^w$  [16], the flow due to transaction  $\omega^w = \{m^w, n^w, t^{w*}\}$  on line  $\ell$  is  $\phi_\ell^w t^{w*}$ . Therefore, the transaction  $\omega^w$  pays the congestion charges  $\lambda_\ell^* \phi_\ell^w t^{w*}$  for the usage of line  $\ell$ . We denote by  $\tilde{\mathcal{L}} \subset \mathcal{L}$  the subset of congested lines:

$$\tilde{\mathcal{L}} = \{\ell \in \mathcal{L} : \lambda_\ell^* > 0\}. \tag{11}$$

The total amount of congestion charges assessed from  $\omega^w$  is

$$\xi^w = \sum_{\ell \in \tilde{\mathcal{L}}} \lambda_\ell^* \phi_\ell^w t^{w*}. \tag{12}$$

The TAP optimality conditions imply

$$\mu_{n^w}^* - \mu_{m^w}^* = \sum_{\ell \in \tilde{\mathcal{L}}} \lambda_\ell^* \phi_\ell^w. \tag{13}$$

Therefore, we may rewrite (12) as

$$\xi^w = (\mu_{n^w}^* - \mu_{m^w}^*) t^{w*}. \tag{14}$$

We denote the optimal value of the social welfare by

$$\mathcal{S}^*|_{TAP} \triangleq \mathcal{S}(p_0^s, p_0^b, \mathbf{p}^s, \mathbf{p}^b, \mathbf{t}^*), \tag{15}$$

where the star and the vertical bar with the TAP subscript indicate that the social welfare is evaluated at the TAP optimum. Now, consider an ideal case where the transmission network has unlimited transfer capability so that all the desired pool and bilateral transactions can be simultaneously accommodated. We define  $\mathcal{S}^*|_u$  to be the maximum social welfare for this transmission unconstrained case. Then, clearly

$$\mathcal{S}^*|_{TAP} \leq \mathcal{S}^*|_u \tag{16}$$

because the network transfer capability limitations may lead to a reduction in the social welfare. We view such a decrease as the costs of congestion. We use the metric  $\mathcal{E}|_{TAP}$  defined to be the decrease in the social welfare resulting from the presence of the transmission constraints

$$\mathcal{E}|_{TAP} \triangleq \mathcal{S}^*|_u - \mathcal{S}^*|_{TAP} \tag{17}$$

to measure the system congestion costs.  $\mathcal{E}|_{TAP}$  is also referred to as the market efficiency loss [5].

Compared to the conventional TSP, the TAP solution may lead to larger social welfare and less market efficiency loss. Note that the

contributions of the bilateral transactions are not explicitly represented in the pool social welfare  $\mathcal{S}(p_0^s, p_0^b, \mathbf{p}^s, \mathbf{p}^b)$  used as the objective function of the TSP. In order to provide a meaningful comparison of the two objective functions, we include the contributions to the social welfare of the bilateral transactions in the desired amount with the TSP solution and so define the total social welfare attained by the TSP solution as

$$\begin{aligned} \mathcal{S}^*|_{TSP} &\triangleq \mathcal{S}(p_0^s|_{TSP}, p_0^b|_{TSP}, \mathbf{p}^s|_{TSP}, \mathbf{p}^b|_{TSP}) + \sum_{w=1}^W \alpha^w(\bar{t}^w) \\ &= \sum_{n=0}^N [\beta_n^b(p_n^b|_{TSP}) - \beta_n^s(p_n^s|_{TSP})] + \sum_{w=1}^W \alpha^w(\bar{t}^w), \end{aligned} \tag{18}$$

where  $p_n^b|_{TSP}$ ,  $p_n^s|_{TSP}$ ,  $n = 0, 1, 2, \dots, N$ , are the optimal values of the TSP. Then, we can compare on a consistent basis the social welfare values of the TAP and TSP solutions. The optimality of the TAP solution ensures that

$$\begin{aligned} \mathcal{S}^*|_{TAP} &= \mathcal{S}(p_0^s|_{TAP}, p_0^b|_{TAP}, \mathbf{p}^s|_{TAP}, \mathbf{p}^b|_{TAP}, \mathbf{t}^*|_{TAP}) \\ &\geq \mathcal{S}(p_0^s|_{TSP}, p_0^b|_{TSP}, \mathbf{p}^s|_{TSP}, \mathbf{p}^b|_{TSP}, \mathbf{t}^*) \\ &= \sum_{n=0}^N [\beta_n^b(p_n^b|_{TSP}) - \beta_n^s(p_n^s|_{TSP})] + \sum_{w=1}^W \alpha^w(\bar{t}^w) = \mathcal{S}^*|_{TSP}. \end{aligned} \tag{19}$$

The inequality in (19) holds since  $(p_0^s|_{TAP}, p_0^b|_{TAP}, \mathbf{p}^s|_{TAP}, \mathbf{p}^b|_{TAP}, \mathbf{t}^*|_{TAP})$ , the optimal solution of the TAP problem in (10), maximizes  $\mathcal{S}(\cdot)$ . We observe that  $\mathbf{t}^*|_{TAP} \leq \bar{\mathbf{t}}$ . The implication is that, under congestion, bilateral transactions that make small contributions to the social welfare may be curtailed in order to provide transmission services to those pool players whose contributions to the social welfare are higher. When no congestion occurs, however, the TAP and TSP solutions are equal and so the equality in (19) holds. Therefore, the social welfare attained by all the transmission customers under the TAP solution is larger than or equal to that under the TSP solution. Such a comparison may also be performed for the congestion costs or the market efficiency loss. We define the market efficiency loss  $\mathcal{E}|_{TSP}$  associated with the solution of the conventional TSP along the same lines used in the definition of  $\mathcal{E}|_{TAP}$ :

$$\mathcal{E}|_{TSP} \triangleq \mathcal{S}^*|_u - \mathcal{S}^*|_{TSP}. \tag{20}$$

From (19), it follows that

$$\mathcal{E}|_{TAP} \leq \mathcal{E}|_{TSP}. \tag{21}$$

That is, in the presence of congestion, the TAP solution results in lower market efficiency loss. Therefore, the TAP solution reflects the more efficient use of the constrained transmission network than that given by the conventional TSP. The ability to appropriately capture this important measure constitutes a key advantage of the TAP formulation.

4. Evaluation of congestion rents and FTR payoffs with the TAP solution

When congestion occurs, the IGO collects the congestion charges from each bilateral transaction in the amount given in (14). The total revenues from these charges are

$$\sum_{w=1}^W \xi^w = \sum_{w=1}^W (\mu_{n^w}^* - \mu_{m^w}^*) t^{w*}. \tag{22}$$

The FERC Order No. 888 [1] comparability provision requires both the pool customers and the bilateral customers to pay congestion charges since the injections/withdrawals of the two classes of customers impact the total congestion costs. In fact, the congestion charges for the pool customers are implicitly included in the LMPs [4,22]. In other words, the pool players' payments for their purchases/sales include the payments for the energy and the implicit

442 payment for the congestion. Absent congestion, the revenues from  
 443 all the pool buyers equal exactly the payments to all the pool sell-  
 444 ers. With congestion, however, the buyers' total payments exceed  
 445 the payments to the sellers. The difference is defined as the *mer-*  
 446 *chandising surplus* [6,7] defined by:

$$\mathcal{K} \triangleq \sum_{n=0}^N \mu_n^*(p_n^{b*} - p_n^{s*}). \quad (23)$$

448 We view  $\mathcal{K}$  as the congestion charges borne by the pool sellers and  
 449 buyers. The congestion rents  $\zeta^{\text{total}}$  are the sum of the implicit con-  
 450 gestion charges assessed from the pool customers and the explicit  
 451 congestion charges paid by the bilateral transactions:  
 452

$$\zeta^{\text{total}} \triangleq \mathcal{K} + \sum_{w=1}^W \zeta^w = \sum_{n=0}^N \mu_n^*(p_n^{b*} - p_n^{s*}) + \sum_{w=1}^W (\mu_{nw}^* - \mu_{mw}^*) t^{w*}. \quad (24)$$

455 In the case of a pool only market

$$\zeta^{\text{total}} = \mathcal{K}. \quad (25)$$

458 The congestion rents are functions of the LMPs, which are highly  
 459 volatile due to the nature of electricity and electricity markets. Such  
 460 volatility introduces uncertainty in the congestion rents. As a re-  
 461 sponse to the transmission customers' needs for risk management  
 462 tools to ensure price certainty in the congestion rents, financial  
 463 instruments known as FTR are introduced. FTR are issued by the  
 464 IGO and entitle the holder to be reimbursed for a share of the con-  
 465 gestion rents collected in the day-ahead markets. The FTR are de-  
 466 fined for a point of receipt (*from* node)  $m$ , a point of delivery (*to*  
 467 node)  $n$ , a specified amount of transmission service  $\gamma$  in MW and  
 468 a per MW premium  $\rho$ . We use the quadruplet

$$\Gamma \triangleq \{m, n, \gamma, \rho\} \quad (26)$$

471 to denote the FTR. The holder of  $\Gamma$  is entitled to receive from the IGO  
 472 a payment of

$$\chi \triangleq (\mu_n^* - \mu_m^*) \gamma, \quad (27)$$

475 where  $\mu_n^*$  and  $\mu_m^*$  are the LMPs as determined in (10). We refer to  
 476 this payment as the FTR *payoff*.

477 Both the congestion rents and the FTR payoffs are functions of  
 478 the LMPs. Therefore, the availability of appropriate LMP values is  
 479 critical for the evaluation of the congestion rents and FTR payoffs.  
 480 The proposed TAP formulation provides a powerful tool for pool-  
 481 bilateral systems to determine the pool market outcomes, the  
 482 transmission service amounts for the bilateral transactions and  
 483 the LMPs. The ability to simultaneously determine the pool out-  
 484 comes and the transmission service provision to the bilateral trans-  
 485 actions makes TAP a highly appropriate vehicle for the evaluation  
 486 of the congestion rents and the FTR payoffs. We next illustrate  
 487 the improvements in the calculations vis-à-vis the conventional  
 488 TSP on various test systems.

489 **5. Simulation study results**

490 We tested the TAP formulation/solution extensively on various  
 491 test systems. We illustrate the quantifiable improvements of TAP  
 492 solutions over those of TSP on these systems including the IEEE  
 493 7-bus, 14-bus, 57-bus and 118-bus networks. The data for these  
 494 test systems are provided in [22]. For each system, we compare  
 495 the TAP and TSP solutions as the volume of the bilateral trans-  
 496 actions is varied. We perform this comparison by evaluating the met-  
 497 rics for congestion rents and FTR payoffs, from both the individual  
 498 player and the system points of view. These metrics include the  
 499 pool market outcomes, the transmission services provided to the  
 500 bilateral transactions, the resulting LMPs and social welfare. We  
 501 discuss some representative results from our studies. The compar-  
 502 ison is measured in terms of the absolute value of the relative devi-

503 ation of the TSP result from that of the TAP normalized with  
 504 respect to the TAP result. For each metric  $q$  of interest, we use  
 505 the error measure  
 506

$$\varepsilon^q \triangleq \left| \frac{q_{\text{TSP}} - q_{\text{TAP}}}{q_{\text{TAP}}} \right|. \quad (28)$$

509 We first compare the TAP and TSP solutions for a specified amount  
 510 of bilateral transactions. We display the results for the IEEE 14-bus  
 511 system in Figs. 1–4. These results are highly representative of all the  
 512 systems tested.

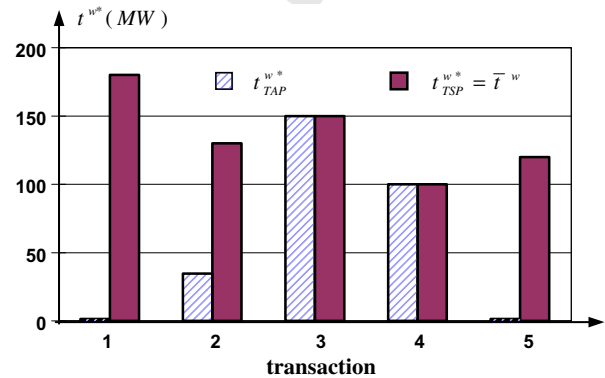


Fig. 1. Amounts of delivered transactions under the TAP and TSP solutions for the IEEE 14-bus system example.

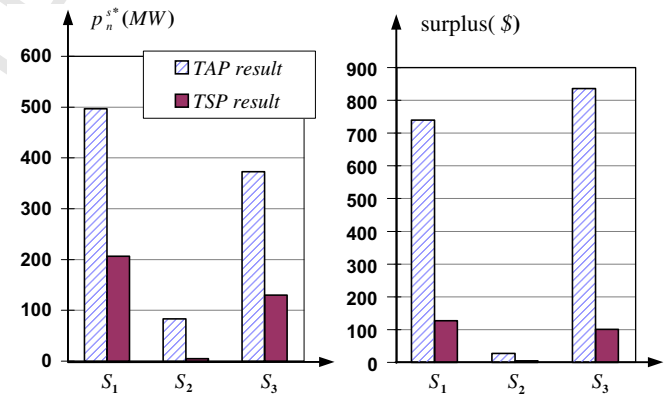


Fig. 2. Sellers' outcomes and corresponding surpluses under TAP and TSP solutions for the IEEE 14-bus system example.

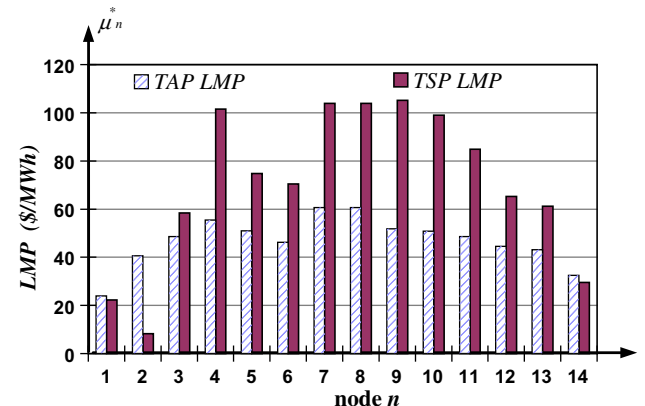
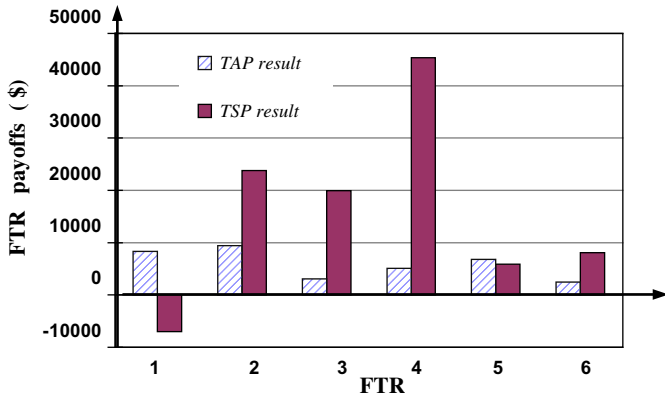


Fig. 3. Comparison of the LMPs computed from TAP and TSP for the IEEE 14-bus system example.



**Table 1**  
FTR holdings for the IEEE 14-bus test system

FTR	From bus	To bus	MW
1	1	2	500
2	1	4	300
3	2	5	300
4	2	10	500
5	1	14	800
6	13	4	200

to portray the integrated decision for the pool and bilateral transaction markets in the proposed TAP solutions.

We next examine the relationships between the market outcomes and the willingness to pay of bilateral transactions under TAP. We focus on transaction 5 and investigate the impacts of increasing its willingness to pay from 15 \$/MWh to 35 \$/MWh. We provide the solutions of the corresponding TAP formulation in Figs. 5 and 6. Fig. 5 shows the amount of transmission services granted as a function of the willingness to pay of the transaction, while Fig. 6 depicts the changes in the LMP differences between the to and from nodes of the transaction as the willingness to pay grows. Clearly, both the transmission services and the LMP differ-

**Fig. 4.** Comparison of the FTR payoffs for TAP and TSP for the IEEE 14-bus system example.

The stark contrast of the TAP and TSP solutions is seen in the plots in Fig. 1 of the transaction amounts for which transmission services are provided under the TAP and TSP formulations. The ability to explicitly consider the interaction between the willingness to pay and the amount of transmission services provided is clearly illustrated for transactions 1, 2 and 5 in the two solutions. Consider the transaction 1 from bus 1 to bus 9 with the willingness to pay function  $\alpha^1(t^1) = 15t^1$ . The TAP solution results in the LMP difference between buses 9 and 1 of

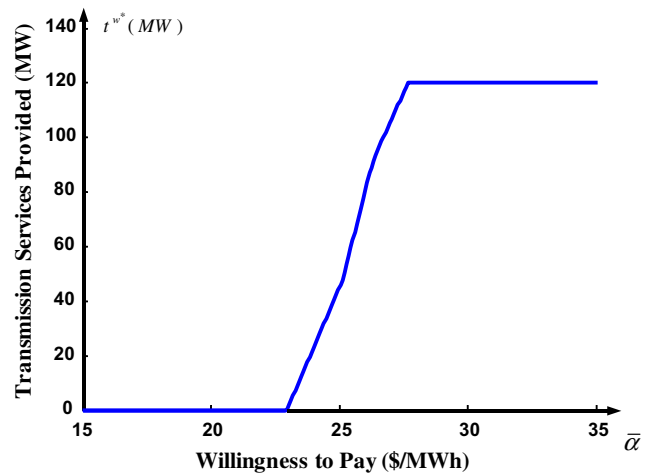
$$50.87 - 23.98 = 26.89 > 15. \tag{29a}$$

Consequently, transaction 1 is entirely curtailed. In the TSP solution, however, this transaction is granted the requested transmission services even though the LMP difference of

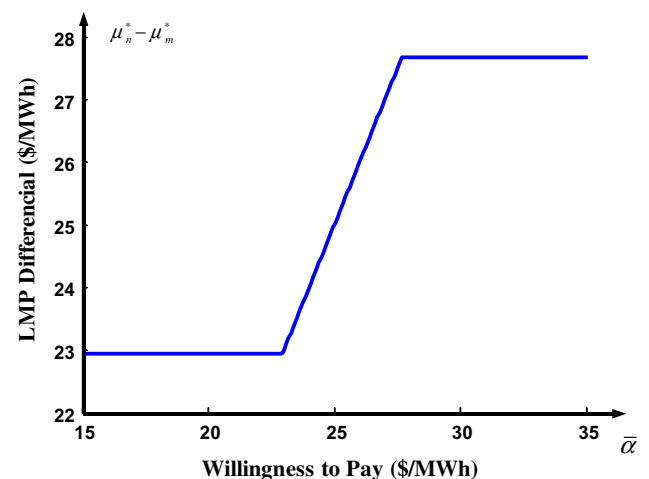
$$98.96 - 22.23 = 76.73 \tag{29b}$$

is considerably larger than the willingness to pay. Similarly, transaction 5 is entirely curtailed for its low willingness to pay. transaction 2 is partly curtailed in the TAP result since its willingness to pay equals the LMP difference between its to and from buses. On the other hand, the transmission services for transactions 3 and 4 are granted in full since the willingness to pay exceeds the LMP differences. The curtailments in the TAP solution correctly reflect the low willingness to pay of the two transactions. This, in turn, results in smaller contributions to the social welfare by the curtailed transactions than those of the pool players. The sellers' sales amounts determined by the two solutions and their corresponding sellers' surpluses are shown in Fig. 2. The two solutions also lead to large differences in the social welfare and the efficiency loss metrics with  $\mathcal{S}^*|_{TAP} = \$32,073$  and  $\mathcal{E}|_{TAP} = \$13,903$  vs.  $\mathcal{S}^*|_{TSP} = \$20,325$  and  $\mathcal{E}|_{TSP} = \$25,651$ . The marked differences in these metrics underline the improvements in market efficiency obtained by the TAP solution.

A direct comparison of the TSP and the TAP LMPs for the IEEE 14-bus system is given in Fig. 3. While the  $\epsilon^{th}$ , the relative error of the LMP at the each bus  $n \in N$  defined using (28), ranges in magnitude from 0.09 to 1.03, the deviations causing these errors impact the market-based settlements for the sales/purchases and also have major repercussions on the congestion rents  $\zeta^{total}$  for the congested network. In the TAP solution, the congestion rents decrease from the TSP solution value of \$ 52,046–\$ 20,799. Such differences are also reflected in the FTR calculations. For the set of FTR holdings given in Table 1, the comparison of FTR payoffs is depicted in Fig. 4. The FTR payoffs errors range from 54% to 158%. In the extensive simulations performed, we found errors to be as large as 500% for some of the test systems. The presence of such large deviations underlines the importance of the capability



**Fig. 5.** Transmission services provided to transaction 5 under TAP as a function of its willingness to pay for the IEEE 14-bus system example.



**Fig. 6.** LMP differences between the to and from nodes of transaction 5 under TAP as a function of its willingness to pay for the IEEE 14-bus system example.

ences are non-decreasing functions of the willingness to pay  $\alpha(\cdot)$ . In this study, we use a simple linear function and vary the value of the parameter  $\bar{\alpha}$ . For the case  $\bar{\alpha} \leq 23$  \$/MWh, the transaction is fully curtailed. When  $\bar{\alpha}$  is between 23 \$/MWh and 27.7 \$/MWh, the transaction becomes marginal and the LMP difference is identical to its willingness to pay, as shown in Fig. 6. The transaction is fully dispatched when  $\bar{\alpha} \geq 27.7$  \$/MWh.

We next illustrate some typical results of the impacts of varying the volume of the bilateral transactions. We introduce a scaling factor  $\zeta$  with which we parameterize the volume of the bilateral transactions. In particular, we multiply the desired transaction amounts  $\bar{t}^w$  of every  $\bar{\omega}^w = \{m^w, n^w, \bar{t}^w\}$ ,  $w = 1, 2, \dots, W$  by  $\zeta$  while we keep the pool players' bids and offers unchanged. We vary the value of the scaling factor  $\zeta$  so that the percentage of the load served by the bilateral transactions increases from 0% to a value at which the TSP no longer has a solution. For each value of  $\zeta$ , we compare the TSP and the TAP solutions and their respective dual variables. Representative plots of the relative deviations obtained from various test systems are presented in Figs. 7–9.

For each of the variables selected, we note the monotonic behavior of the measured deviations. For example, the plots in

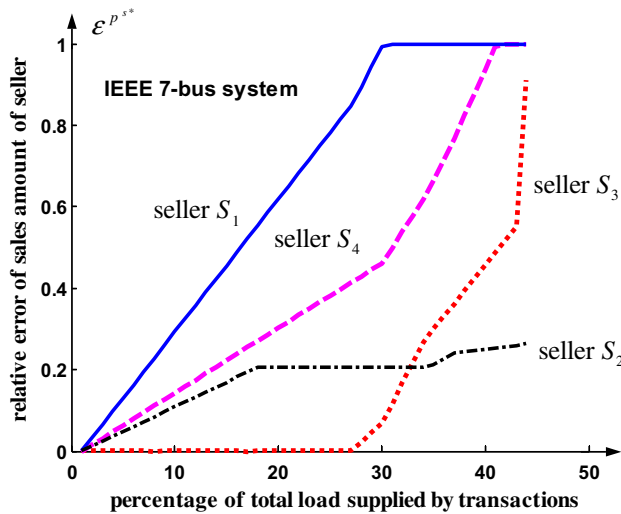


Fig. 7. The relative error of the pool sellers' sales amounts as a function of the bilateral transaction volume in the IEEE 7-bus system example.

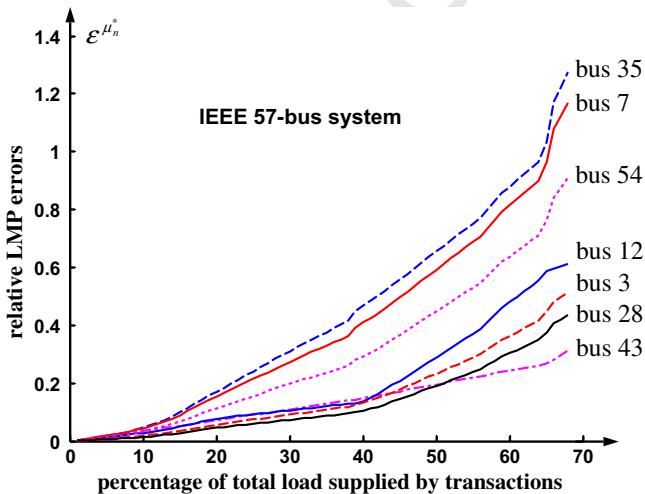


Fig. 8. The relative LMP error as a function of bilateral transaction volume for the IEEE 57-bus system example.

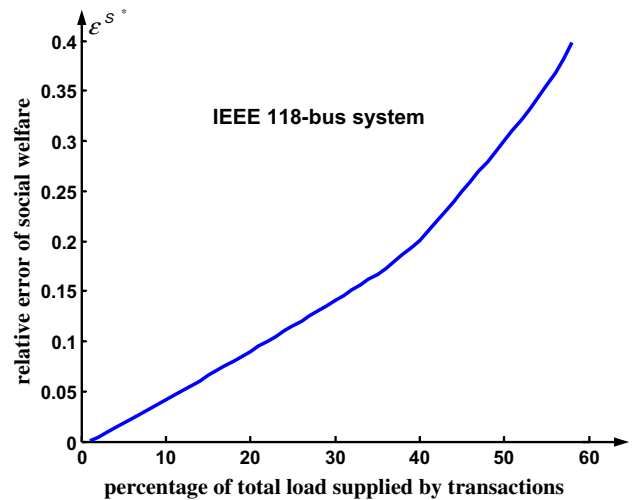


Fig. 9. The relative  $S^*$  error as a function of bilateral transaction volume for the IEEE 118-bus system example.

Fig. 7 illustrate the non-decreasing nature of the differences between the amounts sold by each pool seller in the IEEE 7-bus test system as the scaling factor  $\zeta$  increases. The 100% relative error corresponds to the outcome that the seller  $S_1$  ceases to sell in the pool under the TSP solution since all the limited transmission services are provided to the bilateral transactions. In contrast, under the TAP solution, seller  $S_1$  continues to sell since some transactions are curtailed due to their lower willingness to pay characteristics. We conclude from such studies that the TAP solution provides more realistic outcomes due to the ability to appropriately represent the impact of the bilateral transactions.

The monotonic non-decreasing behavior of the errors is particularly marked for the LMP results. For example, Fig. 8 shows the monotonic behavior in the  $e^{\mu_n}$  as a function of the scaling factor  $\zeta$  for the IEEE 57-bus network. Clearly, for a small value of the scaling factor, the TAP and TSP solutions are very close. As the volume of the bilateral transactions reaches 68% of the total load, the TSP problem becomes infeasible while the TAP solution exists. Thus, the TAP provides the additional advantage of having the ability to determine schedules even in cases where the TSP is no longer solvable. The plot of the relative error of the social welfare in Fig. 9 for the IEEE 118-bus network indicates its monotonic behavior as a function of the scaling factor  $\zeta$ . In addition, we note that the slope of the curve, i.e., the rate of change or the sensitivity of the relative error increases with the scaling factor.

All the plots shown are typical of those observed in the various cases of the test systems studied. These observations indicate the impacts of the integrated pool and bilateral transaction markets decision in the TAP formulation on the LMPs, the transmission scheduling results and, consequently, the congestion rents and the FTR payoffs. The importance of these findings grows as the energy supplied by the bilateral transactions constitutes an increasingly larger portion of the total load in the system.

### 6. Summary

In this paper, we propose an effective tool to compute pool market outcomes and prices in mixed systems where the pool and bilateral transaction paradigms coexist. The proposed tool allows the bilateral transactions to specify their willingness-to-pay for the congestion charges and be provided transmission service accordingly. The interactions between the bilateral customers' willingness-to-pay and their contributions to the social welfare are explored to provide a more comprehensive representation of

the social welfare and market efficiency loss compared to that given by the conventional TSP formulations. The proposed tool is able to provide the more efficient transmission schedules than those given by tools currently used. We illustrated the quantifiable benefits of the proposed scheme on a wide range of systems including the IEEE 118-bus network. The representative numerical results in this paper provide persuasive support for the effectiveness of the proposed formulation and the superiority of its solutions over those of the conventional TSP for the evaluation of congestion charges and the FTR payoffs.

Given the demonstrated advantages of the proposed scheme, a number of extensions are desirable. In this paper, we based our discussions on the lossless DC power flow model. Conceptually, the extension of the formulation to include losses and nonlinear effects by using the full AC power flow is possible. The key challenge to address is the implementation of numerically efficient modifications of the computationally demanding AC power flow model to attain computational tractability of the approach in market applications. Another important extension is the integration of the ancillary service markets for mixed pool and bilateral systems. A critical industry need is transmission investment in the competitive environment. The proposed TAP formulation provides a basis for the study of investment issues such as incentives and transmission adequacy. In addition, the notion of substitutability of transmission and generation for congestion relief is a closely linked topic whose study may be aided by the use of the proposed TAP formulation. Results of our work on these topics will be presented in future publications.

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